

# A Spin Glass Model for Human Logic Systems

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## ABSTRACT

In this paper, we discuss different models for human logic systems and describe a game with nature. Gödel's incompleteness theorem is taken into account to construct a model of logical networks based on axioms obtained by symmetry breaking. These classical logic networks are then coupled using rules that depend on whether two networks contain axioms or anti-axioms. The social lattice of axiom based logic networks is then placed with the environment network in a game including entropy as a cost factor. The classical logical networks are then replaced with "preference axioms" to explore the role of fuzzy logic.

## Categories and Subject Descriptors

J.4 [Social and behavioral sciences]: Economics

## Keywords

complexity, logic, entropy, spin glass

## 1. INTRODUCTION

In this preliminary paper, we discuss the possibilities of interactions between logic systems in complex agents placed in a social network and between individual agents and nature. We examine the effect of uncertainty in measurements by a neural network. First we consider a classical network with a fixed group of axioms. We discuss a quantum neural network that has no fixed axioms, but the probability of using an axiom or its contradiction with certain probability-coefficients in each "decision making measurement". Then we investigate the interactions among these logic networks in a social lattice, and later in a game with nature. We describe the role of entropy in the game with nature, and in interactions with other agents.

## 2. REVIEW OF SYMMETRY BREAKING, ENTROPY AND THE DIRECTION OF TIME

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Entropy was created in nature with symmetry breaking. As the ten-dimensional expanding universe stopped growing in certain dimensions, momentons trapped in the stopped dimensions gave rise to particles in order for their momentum modes to fit within those defined dimension lengths. Entropy was thus created. The second law of thermodynamics states that entropy in a closed system must always increase, attributing to the fact that nature would detest particles clumping together by means of the gravitational force. The gravitational force, on the other hand, is described as the coupling between time and the spatial dimensions, and as the universe expands, the gravitational coupling becomes weaker and weaker, giving rise to increased entropy.

Hence time would become decoupled from space when entropy reaches its maximum.

## 2.1 Information and Entropy

Information may be described as a constraint in nature obtained by symmetry breaking [1]. An increase in information would increase the complexity of the system or an observer in the system because more pieces of information can give rise to more and more laws in which the nature may evolve.

## 2.2 Agents and Information

When an agent obtains a piece of information from nature, it does so by breaking a symmetry. This information gets transformed into an axiom in the agents cognitive network. An increase in the amount of information and hence the number of axioms will imply the possibility of more options by which the agent is able to make a decision by combining the axioms with rules.

## 2.3 Agents as Complex Entities

An agent may be described as a complex meta-stable system that acquires information by interacting with nature.

We propose that agents are connected in social lattices. It is possible to have clusters of social lattices that may later be put into contact with one another. An agent can process any information stored in its cognitive network. Every agent may interact with nature macroscopically. This interaction can be in the form of restructuring the agents environment. Again, any reconstruction of the environment will imply an increase in entropy in nature.

## 2.4 Definitions of Networks

A classical logic network is a network that is based on a fixed set of axioms. The axioms are connected with classi-

cal gates or rules. So given an input, the network uses the axioms and rules to produce an output or a decision. All the axioms and the gates in this model are fixed. We can visualize the scenario by imagining a register representing an axiom. The register holds a value of either 1 or -1 depending on whether the register contains an axiom or the contradiction of an axiom, which we shall refer to as an anti-axiom from now on.

A decision of the network consists of an “action” which may be an interaction with “nature” at a macroscopic level, an interaction with another logic network or an update of a coefficient with which the network is connected to another network (this will be discussed in detail later).

In the quantum version of the network, each register is replaced with two registers that contain the coefficients for the axiom and the anti-axiom,  $a$  and  $b$ , such that  $|a|^2 + |b|^2 = 1$ . However, an agent's final decisions are based on a classical logical semantics where classically defined axioms are connected by rules to produce an output; so that we have a group  $(A = a_1, a_2, \dots, a_n, R = r_1, r_2, \dots, r_n) \Rightarrow D = \text{decision}$ .

During each decision making process, depending on the coefficients, the (axiom + anti-axiom) function collapses to only one of the states, taking the coefficients into account. This type of network would represent a model where a person has a preference, but in different circumstances may choose differently, perhaps based on the other inputs and using rules concerning inputs and coefficients. The improbability of quantum collapse in the human brain has been explored [6]. So we substitute the “conscious” brain network with a model where we introduce “hidden variables.” In this model, a previous decision or an environmental factor triggers the choice of an axiom over an anti-axiom or vice versa by using the coefficient values stored in the twin registers. A detailed model of a hidden variable driven collapse is being developed.

## 2.5 The Classical Logic System and Incompleteness

Gödel's Incompleteness Theorem states that any classical logic system must be incomplete [2]. Classical systems of logic are based on axioms, and the axioms themselves cannot be proven within the logic system.

Now for any axiom in a logic system, we can define an axiom that contradicts it; namely, for  $C$ , we can define  $\text{NOT}(C)$ . Now a classical logical system may not have both  $C$  and  $\text{NOT}(C)$  as underlying axioms, as in that case the system will run into inconsistencies when all the axioms are gated. In other words, there may be no such condition as  $C$  AND  $(\text{NOT}(C))$ . So the logical system chooses either  $C$  or  $\text{NOT}(C)$  as an axiom, and excludes its counterpart.

Now we try to define how quantum uncertainties and symmetry breaking can give rise to logic systems that may be contradictory, or in other words, logical systems that contain contradictory axioms. We propose that nature itself is initially apathetic to  $C$  or  $\text{NOT}(C)$ , or simply, nature does not prefer  $C$  or  $\text{NOT}(C)$ . So we start from a logic vacuum, or a space where for every axiom there is a coexisting anti-axiom. In order to choose which of these clauses should be picked as the underlying axiom, we resort to symmetry breaking. We define states which are mixtures of orthonormal axioms in the same manner we have mixed states in quantum mechanics.

## 2.6 Deriving Classical Logic Systems by Symmetry Breaking

It is possible to imagine a wave function of “axioms” that may be “collapsed” by some measurement. The mode for this “measurement” may be what Roger Penrose describes as “consciousness” [3]. More likely, it is an agent's interaction with nature that defines the rules, or etches the axioms into the neural network. So instead of a “decision space”, we actually go back to a wave particle duality space that establishes “preferences” in the neural network. In other words, we can say that an agent's perception organs acquire these axioms from nature by means of symmetry breaking. Again, each time a measurement is made, a symmetry is broken in the axiom space, so the axiom space is fixed inside a neural network. However, this can be achieved only by disturbing the macro nature, or by increasing entropy in nature. So the axiom space becomes more defined by disturbing the macroscopic nature entropy. We assume that this process of symmetry breaking and acquisition of information is local to the agent and its environment.

## 3. CLASSICAL LOGIC NETWORKS PLACED IN A SOCIAL LATTICE

Now we connect the logic networks in a model akin to the spin model [5]. Here, we can draw similarities with thermodynamics where a micro system (in this case the perceptory organs - coupled to the neural network) is placed in conjunction with a macro system (which is the environment). These logic networks placed in a society may be visualized as a small thermodynamic spin lattice where the axioms residing in different networks are coupled to one another by coupling constants  $J_{ij}$ , and the lattice itself is coupled to a bigger lattice, which is the environment.

However, since we can define the environment lattice to be huge compared to the neural network lattice, we can probably take average values for interaction purposes and couple the lattice with the neural network lattice with some multi-dimensional coupling factor.

In each of the lattices, spins are at a quantum level described as “states” which can coexist in many orthonormal superpositions. However, when the smaller lattice interacts with the bigger lattice, the coupling causes the environment lattice to collapse to a certain value. This value will depend on the probabilistic coefficients of the wave functions and most of the time it would yield the expected value. So an average person will end up with an average set of axioms. Now each of the agents has a certain set of axioms to start with. Again, these agents are coupled with one another in a lattice. A similar model with spin glass models and evolution has been suggested at <http://pespmc1.vub.ac.be/SPINGL.html>. However, we argue that the agents are not connected with one another with a random coupling constant,  $J_{ij}$ , but some rules are defined, and also that these  $J_{ij}$ s are updatable according to the specific state of the entire network.

1.  $J_{ij}$  is not symmetric, i.e.,  $J_{ij} \neq J_{ji}$ . The value of  $J_{ij}$  depends on  $i$  possessing axioms that necessitate the existence of  $j$ . So  $i$  will be coupled to  $j$  more strongly if  $i$  possesses axioms that require the existence of  $j$ .

Now each agent will have the following behaviors in the game:

1. Each agent  $i$  will tend to flip its neighbors axioms if the neighbors axioms contain contradictions of its axioms. The frequency and strength of flipping would depend on a coupling constant  $C_{ij}$ .

$C_{ij}$  depends on the following:

a. The evolution of the logical code developed in  $i$  that contains that particular axiom, i.e. the networking of the certain axiom in the logic network of  $i$ . More accurately, the number of decisions produced by  $i$  that reflect the use of the particular axiom. In other words,  $i$  will tend to flip a neighbors anti-axiom with more effort if the axiom has become an important part of its network, and any future attempt of  $j$ s flipping it would cost  $i$  dearly.

b. The determination of the number of the contradictory axiom in the neighboring agents logic network. An increased frequency of anti-axioms in  $i$ s network would increase the possibility of an anti-axiom to be used in a future decision.

c. The effect of  $j$ s decision in  $i$ s environment (might be caused by physical distance between the two agents)

Now the total strength of coupling between  $i$  and  $j$  ( $i \rightarrow j$ ) would be  $-J_{ij}^n (state_{ni} \rightarrow state_{nj}) + C_{ij}^m (state_{mi} \rightarrow state_{mj})$ ,

where  $n$  and  $m$  are states or registers representing axioms. We can implement this scheme by linking two agents with appropriate gates. The value of this coupling could be described as feelings of agent  $i$  towards agent  $j$ . This total coupling will, at a macroscopic level, cause  $i$  to play for or against  $j$ , i.e. collaborate with  $j$  or work against the existence of  $j$ .

2. An axiom will flip if the effect of the neighbors same axiom state coupling exceeds a flipping energy. The flipping energy depends on:

The certain axioms connectivity with other axioms in the agents cognitive network.

3. All agents must possess an axiom we shall call self preservation or preservation of the network in random probability. We label this axiom  $P$ . Agents containing neither of these axioms cannot contribute to the existence of the agent or to the network. In that case those agents axioms, if few, will be flipped by other agents; or they will self-destroy. A later paper will discuss the effect of agents possessing destructive axioms in their cognitive network, or any critical number that will bound the fraction of agents with self destructive axioms in a network. However, in this paper, we assume that all agents possess  $P$ .

## 4. COMPLEXITY VERSUS ENTROPY

An agent will try to increase its own or the networks complexity and hence stability by the acquired axioms as soon as the acquired axioms get connected to the existence axiom,  $E$ . The agents cognitive complexity increases as it acquires more and more information from nature. This complexity provides the agent with more and more options to create a decision, and hence, increase the entropy of nature.

## 5. INTERACTIONS BETWEEN AGENTS AND NATURE: THE COST FACTOR

Now each social lattice again is coupled to nature, and we can assume the following game being played: The self preservation clause makes an agent play against nature to preserve its own stability while nature tends to increase dis-

order. As disorder increases, so does entropy. The agents play by acquiring information from nature. Axioms added to an agents logical structure contribute to its networks complexity. These new acquired axioms again get entangled with the basic self preservation axiom. This incidence adds more points to the self-preservation side of the game, as now the agents have more rules they can use to preserve themselves.

Now we propose a game with the following rules:

Each agent has an axiom called self preservation or preservation of network chosen at random. We call this axiom the preservation axiom or  $P$ .

The other axioms are acquired by interaction with nature.

Acquiring each axiom has a cost factor  $C$ , since the process requires measurements that increase entropy of the system. The increase of entropy disturbs the meta-stable agent state. The acquired axioms get gated with the preservation axiom and add points to further stabilize the agent. The acquired axioms in different agents are again connected. The contradictory agents are connected repulsively and the supporting axioms attractively.

This game is now being simulated using different values for the couplings constants and the costs. In an earlier work we have presented simulations for a very simple network where agents are connected to neighbors with unitary gates and are allowed to flip when the effects from the neighbors cross a threshold [4].

We add an extra dissipation term with each flip that is linear in entropy, and also a coefficient term with each CNOT gate now chosen at random. In this very simple model, the spins form a closed social network, and we are ignoring the acquisition of new axioms from nature.

## 6. THE DECISIONS

An agent would use a subset of the axioms and rules to produce a decision. The decision might be a macroscopic interaction with the environment. This may lead to restructuring the environment. A restructuring event consists of breaking an organized structure and creating a new one. This action increases the entropy and decreases the total free energy of the system as

$$F = H - TS.$$

Now the interaction might reflect the presence of an axiom by breaking a symmetry in the environment and might shift the expectation value of the axiom from  $\langle A \rangle$  to  $\langle A1 \rangle$ . This implies that another agent seeking an axiom will now have a higher probability of obtaining  $A$  instead of  $\bar{A}$  (Here, by  $A$  and  $\bar{A}$  we mean axiom and anti-axiom). In such a case, an agent possessing  $\bar{A}$  will incur a cost factor because

1. Acquiring  $\bar{A}$  has cost the agent.
2.  $\bar{A}$  might have become connected with its preservation axiom.

The agent possessing  $A$  has two options now:

- a. Inverting  $\bar{A}$  to  $A$
- b. Trying to convert agents holding  $A$

Whether an agent will choose strategy a) or b) will depend on the entropy cost.

## 7. THE PROBABILITY OF OBTAINING AN AXIOM BY ANOTHER AGENT

The probability of obtaining an axiom or an anti-axiom by another agent can now be written down in a simplified version by the formula:

$$F(A) = \sum C_{ijA} f_1(\text{flip}) - \sum J_{ijA} f_2(\text{stabilize}) - f_3(1/R) + K < A >$$

Here,  $A$  is the axiom,  $R$  is a resistance factor for flipping the axiom depending on how entangled the axiom is in the agents own cognitive network,  $K$  is the coupling of the agent with nature, and  $< A >$  is the expectation probability of the axiom in nature. We can see that this formula is very similar to the formula for a classical neural network, except that the coupling constants, unlike the weight factors in a regular neural network, do not sum up to 1. Also, instead of adding a term in  $< A >$ , it might be more realistic to add a term  $F(< A >)$ . Here  $F(A)$  is a switching function that takes on a value of either -1 or 1, depending on whether the RHS exceeds a certain threshold. So every time  $A$  or  $\bar{A}$  flips, the  $C$ s and the  $J$ s are interchanged, and  $R$  is updated to a new value that needs to be updated with the accumulation of new axioms that get entangled with the flipped clause.

A cost of entropy term must be added depending on whether the agent has the take entropy into account axiom or the ignore entropy axiom in its network.

## 8. TRADING AXIOMS

Communicating axioms can be described as a method for obtaining these by several agents at a lower entropy cost than would be necessary in an unsocial (single uncoupled spin) case. The agent obtaining the axiom makes a decision and interacts with its environment macroscopically so that it changes nature to have the information available at a lower entropy cost by other agents.

This can be achieved only when a group of agents have a shared group of rules that relate symbols with possible axioms.

## 9. INTRODUCING FUZZY AXIOMS

A rigid axiom network in each agent will produce a system where too many conflicts are present. We now define a system where the axioms are fuzzy. These fuzzy axioms may be described as preferences.

However, even in this system we initially keep the preservation axiom,  $P$ , non fuzzy or stable. The reason for this formulation is as follows: A probabilistic  $P$  will cut the life-time of the agent, as every collapsed lack of intention to exist decision will work against the existence of the agent. Later in a follow-up paper, we will present a simulation where we see the effect of non-discretizing  $P$ .

So now we have a network with one register holding a value of 1 for  $P$ , and qubits that represent the superposition of both the axiom and the anti-axioms with coefficients  $a$  and  $b$  such that  $|a|^2 + |b|^2 = 1$  for other axioms.

Now, in a logic system where we have one non-fuzzy axiom coexisting with fuzzy axioms, the gates must be designed so that no decision or input can modify the non-fuzzy axiom  $P$ . Hence all the other axioms may be connected in the logic network in a way such that the acquired fuzzy axioms and the decisions or interactions with other agents may modify

the preferences or the coefficients stored in the qubits, but  $P$  stays connected to all other axioms in a way such that no axiom can modify it. The connection path with  $P$  must be one way.

Problems in this scheme arise when two agents are coupled, where the coupling constants between the axiom lattices may tend to modify  $P$  for an agent as both agents try to independently fulfill their axioms keeping only their own  $P$  constant.

### 9.1 Fuzzifying the Existence Axiom

In reality, an agent will interact with other agents, who will not exist forever. So the non-fuzzy I exist" axiom will be coupled with the information that agents die. However, a flip in the I exist axiom as an effect of adding this new piece of information would destabilize the agents cognitive network as any decisions contrary to existence would be self-destructive.

### 9.2 The Spurious Supporting Axioms

An agent possessing  $P$  in combination with the logical clause: "I am an agent and agents die" will be more successful in self-preservation if it can neutralize or alleviate the effect of the latter clause by adding spurious axioms which would contribute a positive factor to  $P$  in order to stabilize it. The other possibility may be living in the present, i.e., ignoring the cost factor of increased entropy and the future.

### 9.3 Conflicting Spurious Axioms

The spurious axioms that would support the "I exist" axiom must be non fuzzy, as fuzzy spurious axioms cannot always lead to the same result, and may contribute negatively to the I exist axiom at times. So for the most efficient stabilization of  $P$ , the designed spurious axioms must be non fuzzy. However, these spurious axioms must be acquired in a fuzzy universe, and may become also be coupled with fuzzy axioms. So agents may again possess a conflicting set of spurious axioms.

### 9.4 The Spurious Axiom Game

The spurious axioms game may be modelled as follows

1. An agent will believe that the spurious axioms are true, and no contradiction may exist.

2. Agents containing a set of spurious axioms conflicting another agents spurious axioms will try to flip the second agents spurious axioms.

3. If flipping is impossible, an agent may try to destroy another agent containing a conflicting spurious axiom as the existence of any conflicting axiom may contradict the agents self existence axiom by coupling.

4. Another strategy would be to keep the spurious axioms private, without showing them to another agent.

### 9.5 Trading Spurious Axioms

An agent unable to maintain a stock of spurious axioms that do not conflict with any other axioms in its cognitive network may buy a spurious axiom from another agent.

The trade may consist of an axiom in exchange of a decision or action by the buying agent. The decision would cost

work from the agent, and hence the agent will lose time and increase the entropy of its own local environment. Agents selling spurious axioms will seek to maximize their gain by selling the axioms to as many agents as possible.

## 9.6 Hiding Spurious Axioms

An agent satisfied with its spurious axiom may hide it, as an open debate might render inconsistencies in it, and hence oblige the agent to buy spurious axioms.

## 10. THE IGNORE ENTROPY AXIOM

Any rule that would ignore the effect of future and entropy would ignore the cost factor in acquiring information, and would make an agent maximize the acquisition of information from nature taking only the increasing complexity factor of the agent into account, and not at the increasing entropy factor of the entire environment. However, the increased entropy of the entire environment again would affect all agents, including those who take the cost factor into account. Since ignore the future agents would destabilize the agents who take entropy into account, a conflict would arise.

## 11. THE GAME OF IGNORE ENTROPY AND A SPURIOUS AXIOM

An agent may play a game by superposing the ignore entropy axiom with a spurious axiom. This may reduce conflicts among spurious axioms. We try to formulate the possible games here and compare.

Agent 1:  $S1$

Agent 2:  $S2$

Agent 3:  $I$

Agent 4:  $a S1 + b I$

Agent 5:  $c S2 + d I$

Agent 6: sell  $S3$  but believe in  $S4$

Agent 7: confused; no axiom

Here  $S1$ ,  $S2$  and  $S3$  are spurious axiom 1, spurious axiom 2 and spurious axiom 3;  $I$  is ignore entropy; and  $a$ ,  $b$ ,  $c$  and  $d$  are coefficients such that in any one superposed wave, the sum of the squares of the coefficients is 1. Also,  $S1$ ,  $S2$  and  $S3$  are normal to one another.  $I$  introduces an extra cost factor  $K(A)$  to other agents with every action made by an agent holding  $I$ . Also, Agent 6 makes a profit of  $K2$  by selling  $S3$  to an agent.  $K2$  is a function of the macroscopic work saved by agent 6. However, Agent 6 must make an investment,  $K4$ , which is a function of the work required to convert the agent. This work might consist of mapping  $S1$  and  $S2$  finding inconsistencies in them.

Now Agent 1 and Agent 2 have two possibilities:

- a. Hide axiom
- b. Show axiom

Besides holding  $S1$  and  $S2$ , Agents 1 and 2 have two other options:

- a. Sell axiom
- b. Optimize on other axioms.

However, in order to sell axiom, agents 1 and 2 must show axiom. So sell axiom must be gated with show axiom with an AND. Now if 1 and 2 optimize on other axioms, they can either show axiom or sell axiom. If they show axiom, 6 will now have its cost function reduced, as 6 now only needs to

find an anti axiom to substitute into 1 and 2's axioms. If 1 and 2 hide axioms, then 6 will have to

- a) Find 1 and 2's axioms, and
- b) Find an anti-axiom.

Now if 1 and 2 decide to show axiom and sell axiom then 1 and 2 will both try to flip each other and 6 will try to flip both 1 and 2. Given that 1 or 2 is able to flip the other and trade axioms, 1 or 2 will profit only when profit made from the other surpasses the effort put into conversion. Now assuming that 1 and 2 both have the same working ability, 1 or 2 will profit only when they have to spend less than half the remaining lifetime of the other trying to flip the other. Now calculating the cost factor and the success of conversion can get very complex, and will be discussed in a later paper.

Now agent 6 will have a strategy to find  $S1$  and  $S2$  properly and design incoherence in them. However, this strategy will succeed depending on being successful achieving the following:

- a. Finding a subgroup of axioms in  $S3$  which agents 1 or 2 may find more indispensable than  $S1$  or  $S2$  and which conflicts with a subset of  $S1$  or  $S2$ .
- b. Convincing agents 1 and 2 that  $S3$  has no logical flaws, based on the other axioms perceived by 1 and 2.
- c. Not assimilating  $S3$  into  $S4$  and creating inconsistencies in its own logical network.

Now any inconsistencies in  $S3$  and the observation of  $S4$  by agent 1 or 2 will lead to disbelief in agent 6. And since  $S1$  and  $S2$  are connected to existence axioms of agents 1 and 2, inconsistencies between  $S3$  and  $S4$  observed by agents 1 and 2 will add a huge cost factor  $K$  to agent 6.

A more detailed simulation of the game is now on the way.

## 12. CONCLUSION

In this preliminary paper we merely suggest some modelling possibilities for development of logical systems when human beings interact. More detailed studies are being carried out.

## 13. ACKNOWLEDGEMENT

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